

Title :

CHANNEL ESTIMATION USING THE GUARD INTERVAL OF A MULTICARRIER SIGNAL

Description**5 Field of the invention**

This invention relates to communication using Orthogonal Frequency Division Multiplexing ('OFDM') and, more particularly, to channel estimation and tracking in OFDM communication.

Background of the invention

10 OFDM communication has been chosen for most of the modern high-data rate communication systems (Digital Audio Broadcast - DAB, Terrestrial Digital Video Broadcast - DVB-T, and Broadband Radio Access Networks - BRAN such as HIPERLAN/2, IEEE802.11a, for example). However, in most cases the receiver needs an accurate estimate of the channel impulse response.

15 In many known OFDM systems, each OFDM symbol of size $N \in \mathbb{N}^+$ is preceded by a guard interval that is longer than the channel impulse response (CIR) and a cyclic prefix of $D \in \mathbb{N}^+$ samples is inserted as the guard interval at the transmitter, the prefix consisting of D samples circularly replicated from the useful OFDM symbol time domain samples. The cyclic prefix enables very simple
20 equalisation at the receiver, where the cyclic prefix is discarded and each truncated block is processed, for example using Fourier Transform (usually Fast Fourier Transform), to convert the frequency-selective channel output into N parallel flat-faded independent sub-channel outputs, each corresponding to a respective sub-carrier. For equalisation purposes, numerous strategies exist.
25 Following the zero forcing approach, for example, each sub-channel output is, unless it is zero, divided by the estimated channel coefficient of the corresponding sub-carrier.

Like other digital communication systems, OFDM modulation encounters problems at high Doppler spreads, which occur notably when the user is moving

fast, for example in a car. HIPERLAN/2, for example, was designed to work only up to speeds of 3 m/s ("pedestrian speed"). Accordingly, the channel impulse response needs to be constantly tracked and updated, especially in the presence of high Doppler spreads.

5 In a known OFDM communication system pilot tones are added which may change their position from one OFDM symbol to another. The amplitudes and positions of the pilot tones are known to the receiver. The receiver uses the pilot tones to estimate the channel coefficients of the corresponding carriers. This method is widely used, but it degrades the system performance, since a certain
10 number of carriers cannot be used for data, since they are reserved for the pilot tones.

It is also known to add learning sequences (See for example EBU Review Technical No. 224, August 1987, "Principles of modulation and channel coding for digital broadcasting for mobile receiver", by M. Alard and R. Lassalle.). In
15 HIPERLAN/2, for example, there are at least 2 learning OFDM symbols per frame (i.e. 2 OFDM symbols of $2 \cdot 4 \mu s$ duration in total per 2ms). If the channel changes quickly, there must be many more training sequences and the consequence is an even bigger degradation in the system performance.

Many of the known systems are unable to decode all carriers of OFDM
20 symbols in the presence of channel nulls. Recent innovations propose ways for decoding OFDM symbols even in the presence of channel nulls (see for example the publication entitled "Reduced Complexity Equalizers for Zero-Padded OFDM transmissions" by B. Muquet, Marc de Courville, G. B. Giannakis, Z. Wang, P. Duhamel in the proceedings of the International Conference on Acoustics Speech and Signal Processing ('ICASSP') 2000 and the publication entitled "OFDM with
25 trailing zeros versus OFDM with cyclic prefix: links, comparisons and application to the HiperLAN/2 system" by Muquet, B.; de Courville, M.; Dunamel, P.; Giannakis, G. in the proceedings of the IEEE International Conference on Communications, 2000, Volume: 2. However, these publications do not offer responses to the
30 problems referred to above concerning channel estimation and channel tracking.

Ideally, the OFDM modulation system would keep all the advantages of classic OFDM and additionally allow very simple and completely blind channel

estimation at the receiver. No additional redundancy would be added to the system and therefore no bandwidth would be lost. Such a system would be advantageous in low-mobility scenarios and would make OFDM systems applicable to high-mobility scenarios as well.

- 5 Many of the examples and illustrations presented below are based on the assumption $N = 4 \cdot D$, that is to say that the size of the prefix (D samples) is assumed to be one quarter of the size of the useful OFDM symbol (N samples). This corresponds to the case of HiperLAN/2 or IEEE802.11. This restriction is introduced for sake of simplicity only. It will be appreciated that the examples and
- 10 illustrations are applicable more generally to the case of $N \in \mathbb{N}^+$, $D \in \mathbb{N}^+$, the necessary adaptation being basically straightforward.

Summary of the invention

The present invention provides a method of, and a transmitter and a receiver for, communication using OFDM as described in the accompanying claims.

15 Brief description of the drawings

Figure 1 is a block schematic diagram of an OFDM communication system comprising a transmitter and a receiver in accordance with one embodiment of the invention, given by way of example,

- Figure 2 is a schematic diagram of an OFDM frame in a signal appearing in
- 20 operation of the system of Figure 1,

Figure 3 is a matrix equation representing the channel impulse response for inter-block interference in operation of the system of Figure 1,

Figure 4 is a matrix equation representing the channel impulse response for inter-symbol interference in operation of the system of Figure 1,

- Figure 5 is a matrix equation representing the combined channel impulse
- 25 response in operation of the system of Figure 1,

Figure 6 is a representation of a sub-matrix corresponding to the combined channel impulse response in operation of the system of Figure 1 for a prefix part of the signal of Figure 2,

Figure 7 represents the upper triangular sub-matrix of the channel matrix presented by Figure 6,

Figure 8 represents the lower triangular sub-matrix of the channel matrix presented by Figure 6 , and

5 Figure 9 is a matrix equation representing signals appearing as a result of the combined channel impulse response in operation of one embodiment of a system of the kind shown in Figure 1,

Figure 10 is a matrix equation representing signals appearing during channel estimation in operation of one embodiment of a system as shown in Figure 1,

10 Figure 11 is a matrix equation representing signals appearing as a result of the combined channel impulse response in operation of another embodiment of a system of the kind shown in Figure 1

Figure 12 is a graph representing preferred values of prefixes used in the system of Figure 1.

15 Detailed description of the preferred embodiments

Figure 1 shows an OFDM communication system in accordance with one embodiment of the invention comprising a transmitter comprising an OFDM modulator 1 and a receiver comprising an OFDM demodulator 2, the transmitter and the receiver communicating over a communication channel 3.

20 An input bit-stream $b_n \in (0,1), n = 0,1,...,K-1$ is modulated onto a set of N carriers whose carrier amplitudes are given by the vector $X(k) = (X_0(k), X_1(k), ..., X_{N-1}(k))^T$, corresponding to OFDM symbol number k . Afterwards, the time domain OFDM signal is generated by means 4 which performs an Inverse Fourier Transform operation, or preferably an Inverse Fast

25 Fourier Transform ('IFFT') operation $[F_N]^{-1} = [F_N]^H$ with $[F_N]^H = ([F_N]^T)^*$ where $(\cdot)^T$ is the transposition operator and $(\cdot)^*$ is the complex conjugate operator:

$$x(k) = [F_N]^{-1} X(k) = (x_0(k), x_1(k), ..., x_{N-1}(k))^T., \quad \text{Equation 1}$$

where $[F_N] = \frac{1}{\sqrt{N}} \cdot (W_N^{lk})_{0 \leq l \leq N-1, 0 \leq k \leq N-1}$, and $W_N = e^{-j\frac{2\pi}{N}}$.

The resulting parallel signal $x(k)$ vector is converted to a series signal by a parallel-to-series converter 5, a prefix, represented by the $D \times 1$ vector $P_D = (c_0, \dots, c_{D-1})^T$, being inserted into the signal as guard interval between each OFDM symbol to produce a series digital signal x_n . The series digital signal x_n is then converted to an analogue signal $x(t)$ by a digital-to-analogue converter 6 and transmitted over the channel 3.

The channel 3 has a Channel Impulse Response $H(k) = C(k)$ and also introduces noise v .

At the receiver 2, an analogue signal $r(t)$ is received and converted to a digital signal r_n by an analogue-to-digital converter 7. The digital signal r_n is then converted to a parallel signal by a series-to-parallel converter $r(k)$ and equalised and demodulated by equalisation and demodulation means 9 to produce demodulated signals $s^{est}(k)$. In the following analysis, consideration of noise is omitted for the sake for simplicity. However, including the consideration of noise does not significantly modify the results.

In some known OFDM communication systems, the guard interval is used to add some redundancy (D samples of redundancy are added) by introducing a cyclic prefix, for example in the following manner:

$$x^{(CP)}(k) = (x_{N-D}(k), \dots, x_{N-1}(k), x_0(k), x_1(k), \dots, x_{N-1}(k))^T.$$

In other words, data from the end of the frame is repeated by the transmitter in the guard interval to produce a prefix.

In accordance with this embodiment of the present invention, however, the prefix samples inserted as guard interval of OFDM symbol number k , $\alpha_k \cdot c_0$ to $\alpha_k \cdot c_{D-1}$, are deterministic and are known to said receiver as well as to said transmitter. The prefixes comprise a vector $P_D = (c_0, \dots, c_{D-1})^T$ of size $D \times 1$ that is common to the symbols multiplied by at least one weighting factor α_k , so that the prefixes have the overall form $\alpha_k \cdot c_0$ to $\alpha_k \cdot c_{D-1}$. The weighting factor α_k may be constant from one

symbol to another. However, in a preferred embodiment of the invention, the weighting factor α_k differs from one symbol to another, the elements of a given vector P_D being multiplied by the same weighting factor. With an OFDM modulator in the transmitter functioning in this way, blind channel estimation in the receiver can be done simply and at low arithmetical complexity. In particular, the receiver can constantly estimate and track the channel impulse response without any loss of data bandwidth. Moreover, the demodulator at the receiver can have advantageous characteristics, ranging from very low arithmetical cost (at medium performance) to high arithmetical cost (very good system performance).

More particularly, in the preferred embodiment of the invention, the prefix of D samples that is added in the guard interval comprises a pre-calculated suitable vector $P_D = (c_0, \dots, c_{D-1})^T$ of D samples that is independent of the data and that is weighted by a pseudo-random factor α_k that only depends on the number k of the latest OFDM symbol:

$$x^{(const)}(k) = (\alpha_k c_0, \dots, \alpha_k c_{D-1}, x_0(k), x_1(k), \dots, x_{N-1}(k))^T. \text{ Equation 2}$$

For the purposes of the analysis below, a second prefix/OFDM symbol vector is defined as follows:

$$x^{(const, post)}(k) = (x_0(k), x_1(k), \dots, x_{N-1}(k), \alpha_{k+1} c_0, \dots, \alpha_{k+1} c_{D-1})^T. \text{ Equation 3}$$

Several choices for α_k are possible. It is possible to choose $\alpha_k \in \mathbb{C}$, that is to say that α_k can be of any complex value. However, any α_k with $|\alpha_k| \neq 1$ leads to performance degradation compared to preferred embodiments of the invention.

It is possible to limit the choice of α_k , somewhat less generally to $\alpha_k \in \mathbb{C}$ with $|\alpha_k| = 1$. This choice usually leads to good system performance, but the decoding process risks to be unnecessarily complex.

Accordingly, in the preferred embodiment of the present invention, the phase of α_k is chosen so that $\alpha_k = e^{j \frac{2\pi}{N+D} m}$, where m is an integer, N is the useful OFDM symbol size and D is the size of the pseudo-random prefix. This choice is

particularly advantageous when using the specific decoding methods described below.

For the sake of simplicity, the following analysis assumes that the weighting factor has been chosen as $\alpha_k = e^{j\frac{2\pi}{N+D}m}$, m integer. However, it will be appreciated
 5 that the mathematical adaptation to any of the cases presented above is straightforward.

It proves to be very useful to choose α_k such that its phase changes from OFDM symbol to OFDM symbol. The constant prefix P_D is preferably chosen with respect to certain criteria, for example the following:

- 10 • In the frequency domain, P_D is as flat as possible over the frequency band used for data carriers.
- In the frequency domain, P_D is as near to zero as possible for all unused parts of the band.
- In the time domain, P_D has a low peak-to-average-power-ratio
 15 (PAPR).
- The length of P_D is the size of the OFDM guard interval, that is to say D samples. Alternatively, a shorter sequence of length $\hat{D} < D$ may be chosen where $D - \hat{D}$ zeros are appended.

With these criteria, without any complication of the transmitter, the receiver is
 20 able to estimate the channel impulse response blindly, track the changes of the channel impulse response blindly and perform an arithmetically simple equalization.

An example of a frame of OFDM symbols in accordance with a preferred embodiment of the invention is illustrated in Figure 2. The operation of the system
 25 will first be described for the specific case where α_k is constant and equal to 1.

Now, the modulation unit of the transmitter is clearly defined. In the following, the operations to be performed in the receiver are considered. Each received OFDM symbol selected at the input of the demodulator 9 can then be expressed as follows (neglecting additive noise):

$$r(k) = [H_{IBI}] \cdot x^{(const, post)}(k-1) + [H_{ISI}] \cdot x^{(const, post)}(k). \quad \text{Equation 4}$$

where the channel impulse response of the demodulator 9 is assumed to be $h = (h_0, \dots, h_{D-1})$, $[H_{IBI}]$ is the contribution of the demodulator 9 channel matrix corresponding to inter-block-interference and $[H_{ISI}]$ is its contribution to inter-symbol-interference.

The components of the received signal corresponding to inter-block-interference $[H_{IBI}]r(k-1)$ are illustrated in Figure 3, where blank elements correspond to zero values, for an example where $N = 4 \cdot D$ (for example, in the case of HiperLAN/2 or IEEE802.11, $N = 64$ and $D = 16$). It will be seen that $[H_{IBI}]$ is a matrix of size $(N+D) \times (N+D)$ with a triangular sub-matrix $[H_1]$ of size $(D-1) \times (D-1)$ at its upper right-hand corner, illustrated by Figure 7, the other elements of the matrix being zero.

The components of the received signal corresponding to inter-symbol-interference $[H_{ISI}]r(k-1)$ are illustrated in Figure 4, for the same case and in the same manner as Figure 3. It will be seen that $[H_{ISI}]$ is a matrix of size $(N+D) \times (N+D)$ with triangular sub-matrices $[H_1]$ on its major diagonal as illustrated by Figure 7 and triangular sub-matrices $[H_0]$ of size $D \times D$ on the diagonal immediately below the main diagonal as illustrated by Figure 8, the other elements of the matrix being zero.

The channel impulse response seen by demodulator 9 is represented by the sum of the inter-block-interference $[H_{IBI}]$ and the inter-symbol-interference $[H_{ISI}]$, as shown in Figure 5. The resulting signal for this example is shown in Figure 9, where $r_0(k)$ to $r_4(k)$ are successive parts of the OFDM symbol # k containing as well contributions of the preceding and following prefix convolved by the channel, $x_0(k)$ to $x_3(k)$ are corresponding parts of size D of the useful signal transmitted and $x_4(k)$ is a corresponding part of size D of the following prefix in this example. Of course, the example may be generalised to any $N \in \mathbb{N}^+$, $D \in \mathbb{N}^+$.

The expectation values of the parts of the received signals are as follows:

$$E_0 = E(r_0) = \underbrace{E([H_0] \cdot x_0(k))}_{=0} + E([H_1] \cdot x_4(k)) = [H_1] \cdot P_D \quad \text{Equation 5}$$

$$E_1 = E(r_1) = \underbrace{E([H_1] \cdot x_0(k))}_{=0} + \underbrace{E([H_0] \cdot x_1(k))}_{=0} = 0 \quad \text{Equation 6}$$

$$E_2 = E(r_2) = \underbrace{E([H_1] \cdot x_1(k))}_{=0} + \underbrace{E([H_0] \cdot x_2(k))}_{=0} = 0 \quad \text{Equation 7}$$

$$E_3 = E(r_3) = \underbrace{E([H_1] \cdot x_2(k))}_{=0} + \underbrace{E([H_0] \cdot x_3(k))}_{=0} = 0 \quad \text{Equation 8}$$

$$5 \quad E_4 = E(r_4) = \underbrace{E([H_1] \cdot x_3(k))}_{=0} + E([H_0] \cdot x_4(k)) = [H_0] \cdot P_D \quad \text{Equation 9}$$

It will be appreciated that the expectation values of the useful parts $x_0(k)$ to $x_3(k)$ of the OFDM symbol tend to zero over a large number of symbols since they are quasi-random with zero mean. However, the prefix P_D is known to the receiver (and in this embodiment is constant over successive symbols) and enables
 10 $[H] = [H_0] + [H_1]$ to be estimated, by approximating the expectation values E_0 and E_4 over a large number R of symbols:

$$E(r) \approx \frac{1}{R} \cdot \sum_{l=0}^{R-1} r(l). \quad \text{Equation 10}$$

The sum of the expectation values E_0 and E_4 is then given by:

$$E_0 + E_4 = ([H_0] + [H_1]) \cdot P_D = [H] \cdot P_D \quad \text{Equation 11}$$

15 A first embodiment of a method of channel impulse response estimation on D symbols in accordance with the present invention utilises the expression of the above equation as follows:

$$\boxed{H_1} \cdot P_D + \boxed{H_0} \cdot P_D = \boxed{H} \cdot P_D = [F_D]^{-1} \cdot \text{diag}(H_0, H_1, \dots, H_{D-1}) \cdot [F_D] \cdot P_D$$

$$\text{Equation 12}$$

where the matrices $[F_D]$ and $[F_D]^{-1} = [F_D]^H = ([F_D]^T)^*$ are the (Fast) Fourier Transform and Inverse (Fast) Fourier Transform matrices respectively and the prefix P_D is of size D . The matrices $[H_0]$, $[H_1]$ and $[H]$ are illustrated by Figure 7, Figure 8 and Figure 6 respectively.

5 Accordingly, in this first method, the channel impulse response is estimated using the following steps:

- Perform a $FFT_{D \times D}$ on $V_{HP} = ([H_0] + [H_1]) \cdot P_D = E_0 + E_4$
- Perform a $FFT_{D \times D}$ on $V_P = P_D$
- Perform a component-by-component division of the first result by the

10 second $\hat{H}_D^{(F)} = V_{HP} \oslash V_P$:

$$\hat{H}_D^{(F)} = FFT_{D \times D}([H_0] + [H_1]) \cdot P_D \oslash FFT_{D \times D}(P_D).$$

- Perform an IFFT on $\hat{H}_D^{(F)}$:

$$\hat{h}_D = FFT_{D \times D}^{-1}(\hat{H}_D^{(F)}).$$

The resulting channel estimation is \hat{h}_D of size $D \times 1$. This method works well
 15 in many circumstances and has a low arithmetic cost, since its calculations are based on matrices of size $D \times D$. However, an OFDM symbol which usually is of size $N > D$ samples will be equalized based on this estimation. Thus, this method works very well if the prefix-spectrum is non-zero everywhere in the $FFT_{D \times D}$ domain (and, of course, everywhere well above channel noise). This can be a
 20 troublesome limitation in other circumstances.

A second embodiment of a method of channel impulse response estimation on D carriers in accordance with the present invention avoids this limitation, at the expense of increased arithmetic cost. This second method does not estimate \hat{h}_D based on a de-convolution in the $FFT_{D \times D}$ domain as presented above, but
 25 estimates $FFT_{(N+D) \times (N+D)}(\hat{h}_D^T 0_N^T)^T$ directly based on the received vector $([H_0] + [H_1]) \cdot P_D$. This is possible by exploiting the observation:

$$[H_{(N+D) \times (N+D)}] \cdot (P_D^T 0_N^T)^T = (E_4^T E_0^T 0_{N-D}^T)^T \quad \text{Equation 13}$$

This equation is represented in more detail in Figure 10. In this second method, the channel impulse response is estimated using the following steps:

- Perform a $FFT_{(N+D) \times (N+D)}$ on $V_{HP} = [H_{(N+D) \times (N+D)}] \cdot (P_D^T 0_N^T)^T$
- Perform a $FFT_{(N+D) \times (N+D)}$ on $V_P = (P_D^T 0_N^T)^T$
- 5 • Perform a component-by-component division $\hat{H}_{N+D}^{(F)} = V_{HP} \oslash V_P$
- If desired, perform an IFFT on $\hat{H}_{N+D}^{(F)}$: $\hat{h}_{(N+D)} = IFFT_{(N+D) \times (N+D)}(\hat{H}_{N+D}^{(F)})$

The last step of the list presented above is not essential for the basic equalization algorithm but may be useful, for example in algorithms used to reduce noise levels.

- 10 The above methods have been described with reference to the specific case where α_k is constant and equal to 1. In preferred embodiments of the invention, however, the weight α_k of the prefix to each symbol k is a preferably complex pseudo-random factor that only depends on the number k of the latest OFDM symbol. The adaptations to this method of the basic equations (shown in Figure 9)
- 15 are shown in Figure 11.

It is found that equations 4 and 8 are to be adapted as follows:

$$E_{\alpha,0} = \underbrace{E([H_0] \cdot r_0(k))}_{=0} + E\left([H_1] \cdot \frac{(\alpha_k \cdot P_D)}{\alpha_k}\right) = [H_1] \cdot P_D. \quad \text{Equation 14}$$

$$E_{\alpha,4} = \underbrace{E([H_1] \cdot r_3(k))}_{=0} + E\left([H_0] \cdot \frac{(\alpha_{k+1} \cdot P_D)}{\alpha_{k+1}}\right) = [H_0] \cdot P_D. \quad \text{Equation 15}$$

- 20 The procedures for blind channel estimation described above remain applicable by setting $E_0 = E_{\alpha,0}$ and $E_4 = E_{\alpha,4}$. This amounts to weighting the preceding and following D prefix-samples of each received symbol by the corresponding α_k^{-1} or α_{k+1}^{-1} respectively.

- 25 The values of the prefixes $\alpha_k \cdot P_D$ are chosen as a function of selected criteria, as mentioned above. Values that have been found to give good results with the criteria:

- Low Peak-to-Average-Power-Ratio of the time domain signal
- Low Out-of-Band Radiation, that is to say maximise the energy of the prefix over the useful band and not waste prefix energy over null carriers
- Spectral Flatness, e.g. SNR of each channel estimates shall be approx. constant
- Low-Complexity Channel Estimation, i.e. by prefix spectrum whose spectral contributions are mainly just phases (i.e. of constant modulus),

are shown in Figure 12 by way of example, for the following OFDM parameters:

- Size of the Prefix in Time Domain: $D=16$ Samples
- Size of the OFDM symbols in the frame: $N=64$ Samples
- Carriers where channel coefficients are to be estimated (over $N+D=80$ carriers): Carriers 1 to 52
- Out-of-Band region: Carriers 76 to 80
- Maximum PAPR has not been limited
- Out-of-Band Radiation as low as possible
- Spectral Flatness as good as possible.

The channel estimation is done by calculating the expectation value over a number of samples of the received vector as explained above. If the tracking of the channel is done based on a first estimation $\hat{h}(k-1) = [F_D]^H \cdot \hat{H}(k-1)$ of the channel impulse response and a number R of OFDM symbols, the first estimate is then updated as follows:

$$\hat{H}(k) = s_0 \cdot \hat{H}(k-1) + \left([F_D] \cdot \sum_{n=0}^{R-1} s_{n+1} \cdot (E_0(k-n) + E_4(k-n)) \right) \oslash ([F_D] \cdot P_D)$$

$$\hat{h}(k) = [F_D]^H \hat{H}(k).$$

based on the ideas of the *first* method for channel estimation that has been presented above. Alternatively, the *second* method can be applied by

$$\hat{H}(k) = s_0 \cdot \hat{H}(k-1) + \left([F_{N+D}] \cdot \sum_{n=0}^{R-1} s_{n+1} \cdot (E_4^T(k-n), E_0^T(k-n), 0_{N-D}^T)^T \right) \oslash ([F_{N+D}] \cdot (P_D^T, 0_N^T)^T)$$

$$\hat{h}(k) = [F_{N+D}]^H \hat{H}(k).$$

where the factors $s_n, n=0,1,\dots,R-1$ are positive real numbers that are used for normalization and weighting of the different contributions. Thus, for example it is possible to take older OFDM symbols less into account for the channel estimation than later ones. The Fourier matrix $[F]$ can be chosen in the $N+D$ carriers or D

5 carriers domain

Several equalization methods are advantageous using the pseudo-random prefix OFDM. In general, the different methods offer different performance-complexity trade-offs.

A first embodiment of a method of equalization uses zero forcing in the $N+D$
10 Domain and offers low complexity equalization.

With $\beta_k = \frac{\alpha_k}{\alpha_{k+1}}$, the Channel Impulse Response matrix can be represented as follows:

$$[H] = [H_{ISI}] + \frac{\alpha_k}{\alpha_{k+1}} \cdot [H_{IBI}] = [H_{ISI}] + \beta_k \cdot [H_{IBI}]$$

$$= \begin{bmatrix} h_0 & \beta_k \cdot h_{N+D-1} & \beta_k \cdot h_{N+D-2} & \dots & \dots & \beta_k \cdot h_1 \\ h_1 & h_0 & \beta_k \cdot h_{N+D-1} & \dots & \dots & \beta_k \cdot h_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ h_{N+D-1} & \dots & \dots & \dots & \dots & h_0 \end{bmatrix} \quad \text{Equation 16}$$

Still assuming that the length of the channel impulse response is D , the
15 coefficients $h_{D+1}, \dots, h_{N+D-1}$ are set to zero. This is a so-called pseudo-circulant matrix, corresponding to the case where β_k is not equal to 1, and can be diagonalized as follows:

$$[H] = \frac{1}{\sqrt{N+D}} \cdot [V]^{-1} \cdot \text{diag} \left(H \left(\beta_k \frac{1}{N+D} \right), H \left(e^{j \frac{2\pi}{N+D}} \cdot \beta_k \frac{1}{N+D} \right), \dots, H \left(e^{j \frac{2\pi}{N+D} (N+D-1)} \cdot \beta_k \frac{1}{N+D} \right) \right) \cdot [V] \cdot \sqrt{N+D}.$$

20 where $[V] = \left(\sum_{n=0}^{N+D-1} |\beta_k|^{\frac{2n}{N+D}} \right)^{\frac{1}{2}} \cdot [F_{N+D}] \cdot \text{diag} \left\{ 1, \beta_k \frac{1}{N+D}, \dots, \beta_k \frac{N+D-1}{N+D} \right\}$

$$\text{and } H(z) = \sum_{n=0}^{N+D-1} z^{-n} \cdot h_n \quad \text{Equation 17}$$

With the assumption that $r(k) = [H] \cdot (x(k)^T P_D^T)^T$, and that the weighting factor has been chosen as $\beta_k = \frac{\alpha_k}{\alpha_{k+1}} = e^{j\frac{2\pi}{N+D}m}$, where m is an integer, and if the received vector $R(k)$ is:

$$R(k) = [H_{\alpha_k, (N+D) \times (N+D)}] \cdot (x(k)^T P_D^T)^T,$$

5 the procedure of this method of zero forcing equalisation is:

- Perform Multiplication $R^{(1)}(k) = \sqrt{N+D} \cdot [V] \cdot R(k)$.

where $[V] = \left(\sum_{n=0}^{N+D-1} |\beta_k|^{\frac{2n}{N+D}} \right)^{\frac{1}{2}} \cdot [F_{N+D}] \cdot \text{diag} \left\{ 1, \beta_k^{\frac{1}{N+D}}, \dots, \beta_k^{\frac{N+D-1}{N+D}} \right\}$,

- Calculate the frequency shifted, estimated CIR coefficients

$$\hat{H}_{N+D}^{\text{Shifted}, F} = \left(\hat{H} \left(\beta_k^{\frac{1}{N+D}} \right), \dots, \hat{H} \left(\beta_k^{\frac{1}{N+D}} \cdot e^{j2\pi \frac{N+D-1}{N+D}} \right) \right).$$

- 10
- Perform a component-by-component division $R^{(2)}(k) = R^{(1)}(k) \oslash \hat{H}_{N+D}^{\text{Shifted}, F}$
 - Perform Multiplication $R^{(3)}(k) = \frac{1}{\sqrt{N+D}} \cdot [V]^{-1} \cdot R^{(2)}(k)$.
 - Extract the N equalized samples of the k th OFDM-data symbol to $S^{EQ}(k)$.
 - Transform the k th OFDM data symbol $S^{EQ}(k)$ into the frequency domain by a Fourier transform $S_F^{EQ}(k) = [F_N] \cdot S^{EQ}(k)$.
- 15
- Proceed with metric calculation, etc. on the received equalised carriers.

Another embodiment of a method of equalization uses a method known from studies on zero padding. The received vector $R(k)$ in the OFDM Pseudo Random Prefix Scheme can be expressed as follows, where $[P]$ contains a $(N+D) \times N$ pre-coding matrix and I_N is the $N \times N$ identity matrix:

$$\begin{aligned} R(k) &= [H] \cdot \left([P] \cdot x(k) + \begin{bmatrix} 0_{N \times N} \\ \alpha_{k+1} \cdot P_D \end{bmatrix} \right) + v \\ &= [H] \cdot \left(\begin{bmatrix} I_N \\ 0_{D \times N} \end{bmatrix} \cdot x(k) + \begin{bmatrix} 0_{N \times N} \\ \alpha_{k+1} \cdot P_D \end{bmatrix} \right) + v \end{aligned} \quad \text{Equation 18}$$

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The Channel Impulse Response estimation obtained as described above is then used together with the known values of P_D to perform the following operation

$$\begin{aligned}
 R^{(1)}(k) &= R(k) - [\hat{H}] \cdot \begin{bmatrix} 0_{N \times N} \\ \alpha_{k+1} \cdot P_D \end{bmatrix} \\
 &= [H] \cdot \begin{bmatrix} I_N \\ 0_{D \times N} \end{bmatrix} \cdot x(k) + v
 \end{aligned}
 \quad \text{Equation 19}$$

in which the known prefix values are multiplied by the Channel Impulse Response estimation and the result subtracted from the received signal. In the general case, $[H]$ is a pseudo circulant channel matrix. So, the diagonalisation of such matrices can then be performed in order to calculate $[H] \cdot P_D$ efficiently. Then, several equalization approaches are possible, for example the Zero Forcing (ZF) approach or the Minimum Mean Square Error (MMSE) equalization approach. Examples of MMSE equalization methods are described in the articles "OFDM with trailing zeros versus OFDM with cyclic prefix: links, comparisons and application to the HiperLAN/2 system" by Muquet, B.; de Courville, M.; Dunamel, P.; Giannakis, G. ICC 2000 - IEEE International Conference on Communications, Volume 2, 2000 and "Reduced Complexity Equalizers for Zero-Padded OFDM transmissions" by Muquet, B.; de Courville, M.; Giannakis, G. B.; Wang, Z.; Duhamel, P. International Conference on Acoustics Speech and Signal Processing (ICASSP) 2000.

In one example, the equalisation is performed based on a zero-forcing approach by multiplying $y^{(1)}$ by the Moore-Penrose pseudo-inverse $[G]$ of the matrix $[H] \cdot \begin{bmatrix} I_N \\ 0_{D \times N} \end{bmatrix}$: $[G] = \left[[H] \cdot \begin{bmatrix} I_N \\ 0_{D \times N} \end{bmatrix} \right]^+$. Thus, the equalized resulting vector is

$$\begin{aligned}
 R^{(eq,ZF)}(k) &= [G] \cdot R^{(1)}(k) \\
 &= [G] \cdot \left[[H] \cdot \begin{bmatrix} I_N \\ 0_{D \times N} \end{bmatrix} \cdot x(k) + v \right]
 \end{aligned}
 \quad \text{Equation 20}$$

The definition of the Moore-Penrose pseudo-inverse is, among others, discussed by Haykin in the book: "Adaptive Filter Theory" by Simon Haykin, 3rd edition, Prentice Hall Information and System Science Series, 1996. Haykin uses the common definition

$$[A]^+ = (A^H A)^{-1} A^H. \quad \text{Equation 21}$$

where $[A]$ is a rectangular matrix.